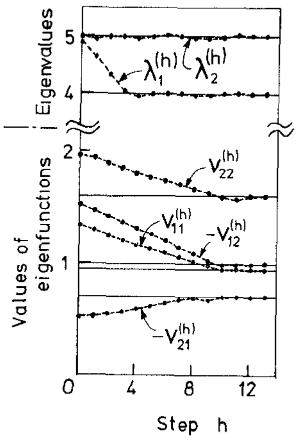


Fig. 10. The behavior of eigenvalues and the values of eigenfunctions at the ports in the case of Fig. 9.

V. Conclusion

A basic algorithm and numerical examples of the synthesis of planar circuits have been presented. When the number of the prescribed eigenvalues and external ports is relatively small, the results are satisfactory both in the computing time and accuracy. However, research is still at a primitive stage; further efforts are needed to make the proposed method practical, especially for larger numbers of eigenvalues and ports. In synthesis processes III and IV, the problem of the multivalent region must be overcome to make those methods practical.

References

[1] T. Okoshi and T. Miyoshi, "The planar circuit—An approach to microwave integrated circuitry," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 245–252, Apr. 1972.
[2] S. Silvester and Z. J. Csendes, "Numerical modeling of passive microwave devices," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 190–201, Mar. 1974.
[3] P. P. Civalleri and S. Ridella, "Analysis of a three-layer rectangular structure," in *Network Theory*, T. Boite, Ed. (Proc. Advanced Study Institute on Network Theory, Belgium, Sept. 1969.) London, England: Gordon and Breach, 1972.
[4] T. Okoshi and M. Saito, "Analysis and design of distributed planar circuits," presented at 1974 IEEE Int. Conf. Circuits and Systems, Apr. 23–25.
[5] T. Okoshi, Y. Uehara, and T. Takeuchi, "Segmentation method—An approach to the analysis of microwave planar circuit," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 662–668, Oct. 1976.
[6] K. Grüner, "Method of synthesizing nonuniform waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 317–322, Mar. 1974.
[7] A. Ben-Israel, "A Newton–Raphson method for the solution of systems of equations," *J. Math. Anal. Appl.*, vol. 15, pp. 243–252, 1966.
[8] O. Yu. Kul'chitskii and L. I. Shmelevich, "Determination of the initial approximation for Newton's method," *USSR Comp. Math. Phys.*, vol. 14, no. 4, pp. 188–190, 1974.

Equivalent Circuit Capacitance of Microstrip Step Change in Width

CHANDRA GUPTA, STUDENT MEMBER, IEEE, AND ANAND GOPINATH, MEMBER, IEEE

Abstract—**Calculated results which extend existing data on the capacitance of step discontinuity are presented for w_1/H of value 0.1, 0.5, 1.0, and 2.0, for relative dielectric constants of 15.1, 9.0, 4.0, and 2.3, and for w_2/H in the range 0.1–10.0. The quasi-static method of calculation is used, and the excess capacitance associated with the steps is determined by the solution of the integral equation using Green's functions.**

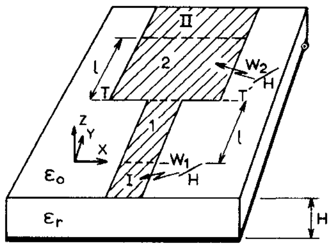
Introduction

THE RANGE of data currently available on the microstrip discontinuities is inadequate, thus microstrip circuit designs are currently implemented after a few trial stages. The present paper extends the range of the capacitances of the step change in width discontinuity beyond that provided by Farrar and Adams [1] and Benedek and Silvester [2]. The calculations performed for this data utilize the integral equation approach using Green's functions and the concept of "excess charge" due to Benedek and Silvester [2] to preserve numerical accuracy. The method of solution discretizes the discontinuity into rectangular elements and the excess charge is obtained by the Galerkin method.

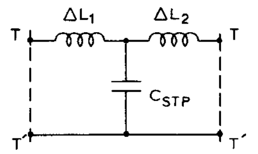
Radiation and dispersion effects are neglected, and, therefore, the microstrip discontinuity problem may be reduced to a quasi-static form. The stored energy of the step discontinuity may then be represented by an equivalent circuit in the form of a T circuit, given in Fig. 1(b), for the chosen reference planes TT'. The present calculations evaluate the shunt capacitance of this circuit, the inductive components have been presented elsewhere [3].

In the following sections, we briefly outline the formulation and the method of solution followed by the results.

Manuscript received December 10, 1976; revised March 1, 1977. This work was supported by the U.K. Ministry of Defence, Procurement Executive under the CVD Directorate and is published with permission. C. Gupta is supported by a UCNW scholarship.
The authors are with the School of Engineering Science, University College of North Wales, Bangor, Gwynedd LL57 1UT, U.K.



(a)



(b)

Fig. 1. (a) Step in a microstrip. H is the height of the substrate. ε_0 is the permittivity of free space; ε_r is the permittivity of the dielectric. I and II are infinite lines terminating at TT'. 1 and 2 are the patched rectangles to calculate "excess" charge. (b) Equivalent circuit for the step discontinuity. C_{STP} represents the capacitance associated with the step discontinuity. ΔL_1 and ΔL_2 are the inductances associated with the discontinuity.

Formulation and Method of Solution

The method of solution is to assume that the infinite-line charge distributions extend from $\pm\infty$, respectively, to the reference plane TT' in Fig. 1(a). These charge distributions are estimated for the infinite line, assuming constant potential in the normal way, using the concept of partial images in the substrate [4]. However, these lines are semi-infinite lines terminating at the reference plane, and to maintain potential constant through the junction, additional "excess" charge has to be added on these infinite-line charges. Thus this excess charge is approximated over rectangular elements patched on to the semi-infinite lines by expansion in a trial set of functions. The coefficients of this expansion are determined by the requirement that the potential over all regions on the strip including the discontinuity remains at the constant value.

Thus we have with the observation points on rectangular element 1:

$$\int_1 G_{11}\sigma_{e1}\,dS + \int_2 G_{12}\sigma_{e2}\,dS + \int_I G_{-\infty/2}\sigma_{\infty I}\,dl + \int_{II} G_{+\infty/2}\sigma_{\infty II}\,dl = \phi \quad (1)$$

where

- σ_{ei} the unknown excess charge distribution in the ith rectangular element;
- $\sigma_{\infty J}$ the known infinite-line charge distribution in the Jth line for potential ϕ;
- G_{ij} the Green's function with the observation point in the ith rectangle due to excess charge in the jth rectangle (see (A1) in the appendix);
- $G_{\pm\infty/2}$ the Green's function for the semi-infinite line extending from the reference plane TT', $y = 0$ to $y = \pm\infty$ (see (A2) in the appendix);
- ϕ potential on the whole strip structure.

Solution of (1) provides the unknown charge distribution σ_{e1}, and the discontinuity capacitance to ground is given by

$$C_{\text{STP}} = \left[\sum_{i=1}^{2} \int_i \sigma_{ei}\,dS\right]^2 \Big/ \int_S \sigma_{ei}\phi\,dS. \quad (2)$$

We approximate σ_{ei} in a trial set of functions $f_j(x,y)$ with the approximation valid only over the ith rectangle:

$$\sigma_{ei} = \sum_{j=1}^{n} a_j^i f_j(x,y). \quad (3)$$

Taking the inner product of each term in (1) with $f_j(x_0,y_0)$, gives n equations. Repeating this with the observation points in all i rectangles, gives the matrix equation from which all the coefficients a_j^i are uniquely determined. Subsequent integration and summation as in (2) determines the required equivalent circuit capacitance.

In these calculations for the step capacitance, the trial set of functions comprised of Newton–Lagrange equispaced interpolatory polynomials. Integrations were performed using Gaussian quadratures with appropriate numbers of points. In the case of a singular integral where the observation points and expansion set lie in the same rectangle, a change of variables eliminates this singularity [5] and then integration is carried out using Gaussian quadratures.

Results

The infinite-line charge distributions were estimated using eleventh-degree equispaced interpolatory polynomials, with an adequate number of quadratures in this case to provide accurate solutions. Comparison of the infinite-line capacitance with other results in the literature [6], [7] shows negligible differences in all cases.

Tests on the capacitance of the step discontinuity were carried out by varying the degree of polynomials in x and y from the fifth to the eighth degree, and on different lengths of rectangles from $l = H$ to $3w$. These results show at worst a variation of less than 3 percent over the whole range; therefore, all results were calculated for $l = 3H$, with polynomials of the fifth degree in x and y. The choice of this degree of polynomials was dictated by computer time considerations, which became excessive for higher degrees.

The step results obtained for $w_1/H = 1.0$, $H = 1.0$ m, for various w_2/H substrates $\varepsilon_r = 9.6$ are compared with those of Farrar and Adams [1] and Benedek and Silvester [2] in Table I. This table also gives the results of our calculations obtained when the trial functions are bivariate monomials complete to the second degree, i.e., $[1, x, y, x^2, xy, y^2]$, and when the degree of the interpolatory polynomials is the fifth degree in x and y. In all cases the results of the interpolatory polynomials is marginally higher than those given in [2], though in any experimental verification these differences will be lost in the uncertainty of the measurements. It is thought that the results of the interpolation polynomials are more accurate since their degree is higher and also they are vectorially better conditioned.

Fig. 2(a)–(d) provides, with curves of the equivalent circuit, the normalized capacitance obtained from our com-

TABLE I

w_2/H	Farrar and Adams [1]	Benedek and Silvester [2]	Step Equivalent Capacitance Calculated in Picofarads	
			Monomial Set $[1,x,y,x^2,xy,y^2]$	Newton–Lagrange Equispaced Interpolatory Polynomials
2.0	14.0	14.1	13.94408	15.14017
3.0		39.9	41.49674	43.62969
4.0		76.0	74.35829	76.60971
5.0		110.0	109.4661	112.5200
6.0		144.0	146.3523	151.8152

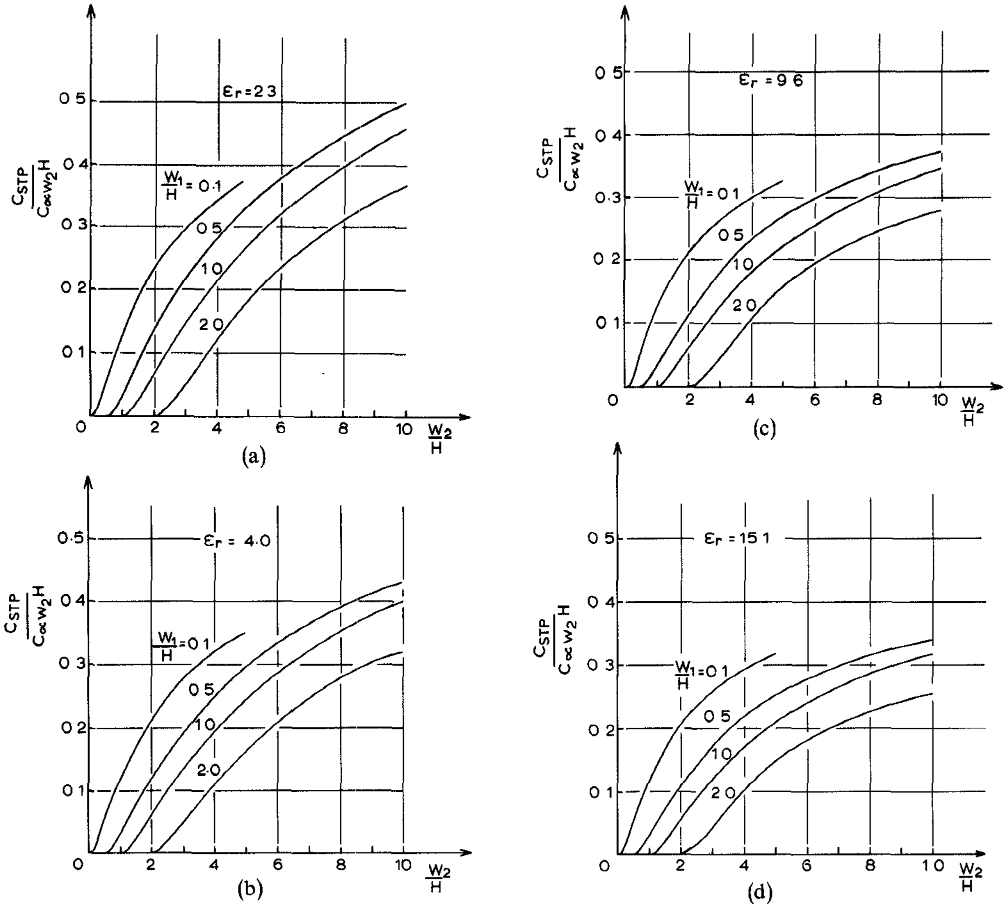


Fig. 2. (a)–(d) Step capacitance against w_2 using interpolation polynomials. $C_{\infty w2}$ capacitance per unit length of line w_2.

puter program, for $\varepsilon_r = 2.3$, 4.0, 9.6, and 15.1; for $w_1/H = 0.1$, 0.5, 1.0, and 2.0; and for various w_2/H. These calculations have been carried out using fifth-degree (in x and y) equispaced interpolation polynomials.

Appendix

The Green's function for the rectangular regions patched on the semi-infinite lines with unit charge distribution is

$$G_{ij}(x,y,H/x_0,y_0,H) = \frac{(1-k)}{4\pi\varepsilon_r\varepsilon_0}\left[\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2}} - (1-k)\sum_{n=1}^{\infty}\frac{k^{n-1}}{\sqrt{(x-x_0)^2+(y-y_0)^2+4(nH)^2}}\right]. \quad \text{(A1)}$$

The Green's function for the semi-infinite line I with unit charge distribution is

$$G_{-\infty/2}(x,y,H/x_0,0,H) = \frac{(1-k)}{8\pi\varepsilon_r\varepsilon_0}\left[-\log_e\frac{\sqrt{(x-x_0)^2+y^2}+y}{\sqrt{(x-x_0)^2+y^2}-y} + \sum_{n=0}^{\infty}k^n\log_e\frac{(x-x_0)^2+4(n+1)^2H^2}{(x-x_0)^2+4n^2H^2} + (1-k)\log_e\frac{\sqrt{(x-x_0)^2+y^2+4n^2H^2}+y}{\sqrt{(x-x_0)^2+y^2+4n^2H^2}-y}\right] \quad \text{(A2)}$$

where

k $= (\varepsilon_0 - \varepsilon_r\varepsilon_0)/(\varepsilon_0 + \varepsilon_r\varepsilon_0)$,

H height of the substrate,

x,y,H are the observation points on the strip,
x_0,y_0,H are the source points on the strip.

Acknowledgment

Thanks are due to Prof. I. M. Stephenson, B. Easter, and B. M. Neale for advice and suggestions.

References

[1] A. Farrar and A. T. Adams, "Matrix method for microstrip 3-d problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 497–503, Aug. 1972.
[2] P. Benedek and P. Silvester, "Equivalent capacitances for microstrip gaps and steps," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 729–733, Nov. 1972.
[3] A. F. Thompson and A. Gopinath, "Calculation of microstrip discontinuity inductances," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 648–655, Aug. 1975.
[4] P. Silvester, "TEM wave properties of microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*; also *Proc. Inst. Elec. Eng.*, vol. 115, no. 1, pp. 43–48, Jan. 1968.
[5] P. Silvester and P. Benedek, "Capacitance of parallel rectangular plates separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 504–510, Aug. 1972.
[6] ——, "Electrostatics of the microstrip-revisited," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 756–758, Nov. 1972.
[7] G. M. L. Galdwell and S. Coen, "A Chebyshev approximation method for microstripline problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 865–870, Nov. 1975.

Equivalent Series Inductivity of a Narrow Transverse Slit in Microstrip

WOLFGANG J. R. HOEFER, MEMBER, IEEE

Abstract—**The series inductivity introduced by a narrow transverse slit in a microstrip transmission line has been evaluated theoretically, and a simple formula for this inductivity is presented. Experimental results for slits of different depth obtained with the resonant ring method compare well with theory. Applications of such a slit include the fine tuning of the electrical length of stubs and the compensation of excess capacitances at discontinuities.**

I. Introduction

THE INSERTION of a narrow transverse slit or notch into a microstrip transmission line leads to a local concentration of the magnetic field which can be described in terms of an equivalent series inductivity. Fig. 1(a) shows a microstrip of width w containing a slit of depth a and width b, centered about the $z = 0$ plane. Some current lines and lines of magnetic field have been drawn to demonstrate the effect of the slit on the propagating quasi-TEM fields.

The equivalent series inductivity ΔL (Fig. 1(b)) is independent of the slit width b as long as the slit is narrow, i.e., b is smaller than the substrate thickness h and much smaller than the transmission line wavelength λ_g. In the following, the normalized series inductivity $\Delta L/h$ will be calculated as a function of w/h of the microstrip and the relative depth a/w of the slit. It is assumed that the substrate is nonmagnetic ($\mu_r = 1$). Dispersion and capacitive effects will be neglected.

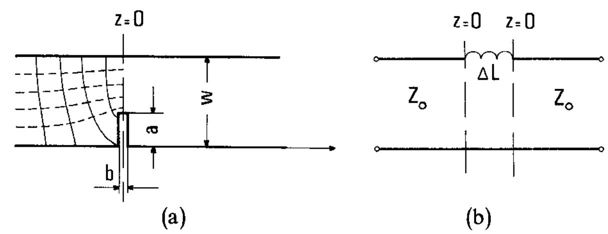


Fig. 1. (a) Narrow transverse slit in a microstrip line. Some current lines (dotted) and magnetic field lines (solid) demonstrate the effect of the slit. (b) Equivalent series inductivity in the $z = 0$ plane.

Manuscript received October 15, 1976; revised March 24, 1977.
The author was with the Institut National Polytechnique, Laboratoire d'Electromagnetisme, E.N.S.E.R.G., Grenoble, Cedex, France. He is now with the Department of Electrical Engineering, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5.

II. Analysis of the Slit

The parameters of many transmission-line discontinuities can be calculated with reasonable accuracy by assuming that they create a perturbation in the form of a dipole field when excited by the incident field. Wheeler [1] has outlined some basic principles of this method. His equivalent volume concept will be applied in the following study.

The main difficulty in the calculation of the slit inductivity resides in the fact that, on the one hand, the application of the equivalent volume concept calls for a uniform excitation of the slit, but, on the other hand, the slit is situated partly in the highly nonuniform fringing field of the microstrip.

To overcome this problem, the microstrip line is replaced